Bayesian Demographic Estimation and Population Reconstruction

UN Population Division Expert Group Meeting on *Methods for the World Population Prospects 2021 and Beyond*

6–8 April 2020

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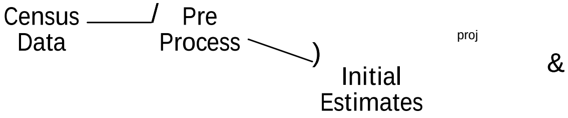
Acknowledgements: Joint work with Adrian Raftery, Sam Clark and Patrick Gerland
NICHD, Grants R01 HD054511 and K01 HD057246
BayesPop Working Group, CSSS, and CSDE at the UW
FHES at AUT

WPP Estimation Process



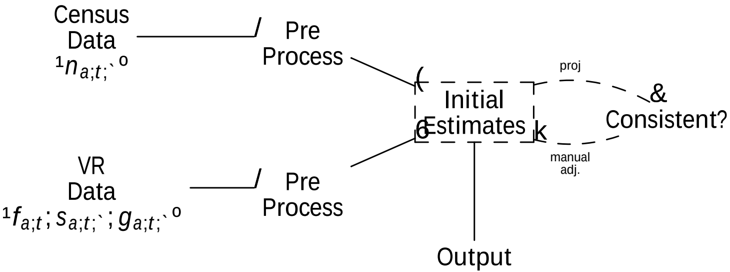
Bayesian Population Reconstruction

Bayesian Population Reconstruction was proposed as an improved method for reconstructing populations of the recent past.

Goals

1. Quantify uncertainty probabilistically.
2. Estimate all parameters consistently.
3. Be easily replicable.
4. Use all reliable data and expert opinion.

Current UN Practice



Bayesian Population Reconstruction

Parameters and Notation

Indices

Age groups »*a*; *a* ý †º, *a* = •; : : : ; ˆ†, »‰•; 1º.

Time periods »*t*; *t* ý †º, *t* = *t*•; : : : ; *T* †; *t*• is the “baseline” year.

Sex `, *F*, *M*.

Parameters

Hierarchical Model

(= initial estimates and census counts which are fixed)

I. Likelihood

$$\log n_{a;t} \; j \; n_{a;t}; \; {}^{f}_{n} \quad \text{Normal} \; \log n_{a;t}; \; {}^{f}_{n}$$

II. Projection Model

$$n_{a;t} \; j \; n_{t\ \dagger}; f_{t\ \dagger}; s_{t\ \dagger}; g_{t\ \dagger} = CCMPP^{1} n_{t\ \dagger}; f_{t\ \dagger}; s_{t\ \dagger}; g_{t\ \dagger}{}^{o}$$

III. Priors on Inputs

$$\log SRB_t \; j \; SRB_t; \; {}^{f}_{SRB} \quad \text{Normal} \; \log SRB_t; \; {}^{f}_{SRB}$$

$$n_{a;t}. \quad \text{Unif} \; \bullet; K \quad ; K > \bullet$$

$$\log f_{a;t} \; j \; {}^{f}_{f} \quad \text{Normal} \; \log f_{a;t}; \; {}^{f}_{f}$$

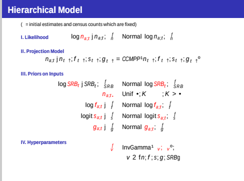
$$\text{logit} \; s_{a;t} \; j \; {}^{f}_{s} \quad \text{Normal} \; \text{logit} \; s_{a;t}; \; {}^{f}_{s}$$

$$g_{a;t} \; j \; {}^{f}_{g} \quad \text{Normal} \; g_{a;t}; \; {}^{f}_{g}$$

IV. Hyperparameters

$${}^{f}_{v} \quad \text{InvGamma}^{1} \; {}_{v}; \; {}_{v}{}^{o};$$

$$v \; 2 \; \text{f} n; f; s; g; SRB \text{g}$$



The model has four levels. I shall start at level 3:

3 The true vital rates and baseline population are given priors, conditional on the initial estimates and variance parameters which account for measurement error.

4 The variance parameters are, themselves, given distributions at level 4.

2 The priors on the vital rates and baseline counts induce a prior on subsequent counts through the deterministic projection model in level 2.

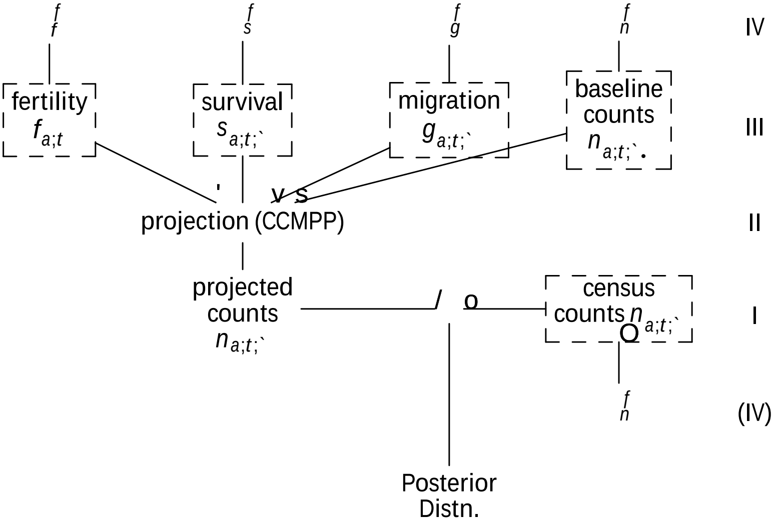
1 Level 1 is a likelihood for the census counts which is parameterized by the projected counts and a variance parameter which accounts for measurement error.

1. Here are the things we have data on
2. Here are the things we have to specify
3. Here are the things we do inference on

Inference

Hierarchical Model: Key Relationships

(inputs are boxed)



Measurement Error Variance

$$ {}^{f}_{v} \quad \text{InvGamma}\left({}_{v};\ {}_{v} \right);\ v = f;\ s;\ g;\ n; $$

Measurement Error Variance

$\sigma^2_v \sim \text{InvGamma}(\alpha_v, \beta_v); \ v = f, s, g, n, SRB$

The σ^2_v are random. They reflect non-systematic error in the initial estimates.

We specify α_v and β_v using expert opinion
- possibly informed by empirical estimates of measurement error,
- or elicited as mean absolute relative error (MARE) through statements like

"With probability 90 percent, the initial estimates are accurate to within approximately p percent."

This is a flexible approach: some data sources have information about their accuracy, but others do not.

We choose the hyperparameters using the MAE of the marginal prior distributions of the observables. For example, we use the joint marginal distribution of the vector of fertility rates, $(f_{,,,}, \ldots, f_{A,T})$ which is

$$t_{AT} \ \log f \ ; \ _f \bullet \ _f \mathbf{I}_{AT}; f \quad ; \qquad , \bullet f; \quad > \bullet$$

Then

$$\text{MAE}(\log f) \quad \text{E} \, | \log f \quad \log f \ |$$
$$= \text{E} \ \text{E} \Big[\, | \log f \quad \log f \ | \quad f \, \Big]$$
$$= {}_{\prime AT} \ \text{E} \left(\sqrt{\frac{}{f}} \ f \right)$$
$$= {}_{\prime AT} \ \sqrt{\frac{}{f}} \ \underbrace{\frac{1 \qquad , \bullet f)}{1 \quad)}}_{h(\quad)} ; \qquad , \bullet f; \quad > \bullet$$

where ${}_{AT}$ is a length AT vector of 1s.

Example: Laos and New Zealand

The method is most useful for countries with unreliable, fragmentary data (high uncertainty).

However it also works for countries with very good data.

Reconstructing the female populations of Laos (1985–2005) and New Zealand (1961–2006)

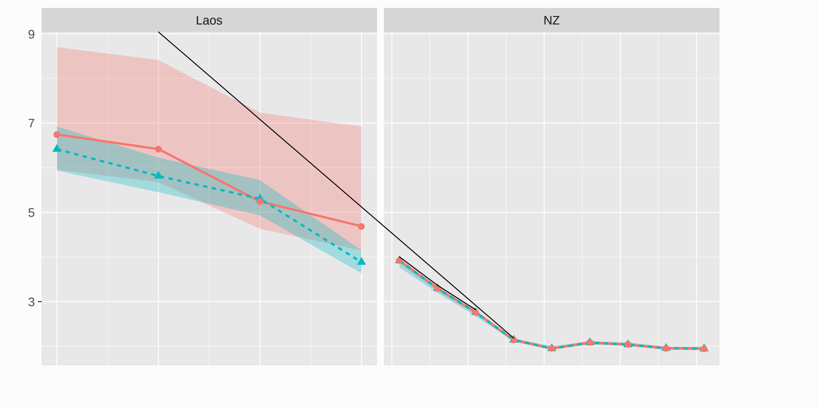
The reconstruction periods are determined by the available data.

Population sizes are similar (counts in millions):

	1961	1985	2005
Laos	—	1.8	2.8
New Zealand	1.2	1.7	2.1

Data quality and availability are very different.

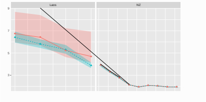
Total Fertility Rate



95% Posterior Interval Half-widths

	Mean Half-Width
Laos	0.4
NZ	0.04

Total Fertility Rate



95% Posterior Interval Half-widths

	Mean Half-Width
Laos	0.4
NZ	0.04

- These plots show prior and posterior medians and the limits of 95% credible intervals.
  - The prior on TFR is in red, the posterior is in blue.
  - The initial estimates correspond to the prior median, shown by the red line.
- TFR in Laos was much higher than in New Zealand.
- Posterior uncertainty is quantified using the mean half-width of the credible intervals:
  - Mean half-width for New Zealand was about 1/10th that of Laos.
- I show the WPP 2010 estimates for comparison

Total Net Migration

95% Posterior Interval Half-widths

	Counts (000s)
	Mean Half-Width
Laos	9.5
NZ	2.6

Summary

Inputs

Use in WPP

- Computation speed needs to be fast—could be a challenge with single-year age and time.
- Needs to expand beyond span of most census collections, back to 1950.
  - Some countries have only one or two censuses, or rely on admin. data, household surveys.
- Inputs and/or measurement error could be modelled (but see first point).
- Extensive testing required.

References

Wheldon, M. C., Raftery, A. E., Clark, S. J., and Gerland, P. (2013),